RADIUS OF A TUBE WITH HEAT SOURCES (SINKS)

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The radius of a tube is determined of the permissible temperature drop across the tube section.

When a tube is designed for a reactor with a granular bed of catalyst, then the permissible temperature drop across the tube section determines the radius. The temperature profile across a tube section is, in the case of a zeroth-order reaction, analogous to that in [1]:

$$\pm\theta = \ln\frac{8}{\delta\left[\exp(-\mu)\rho^2 \pm \exp\mu\right]^2}, \quad (1)$$

where $\exp(-\mu) = \sqrt{\sigma}$, and σ are the roots of the characteristic equation

$$\frac{8\sigma}{\delta(\sigma \pm 1)^2} = \exp\frac{4\sigma}{\mathrm{Bi}(\sigma \pm 1)}. \quad (2)$$

Here and later on the upper sign refers to an exothermal reaction, the lower sign refers to an endothermal reaction.

The method proposed in [1] is convenient to use for determining the permissible tube radius on the basis of the given temperature drop from the center to the wall. From expression (1) we have

$$\Delta\theta = |\theta_c - \theta_w| = 2\ln(\sigma \pm 1), \quad (3)$$

$$\sigma = \exp(\Delta\theta/2) \mp 1. \quad (4)$$

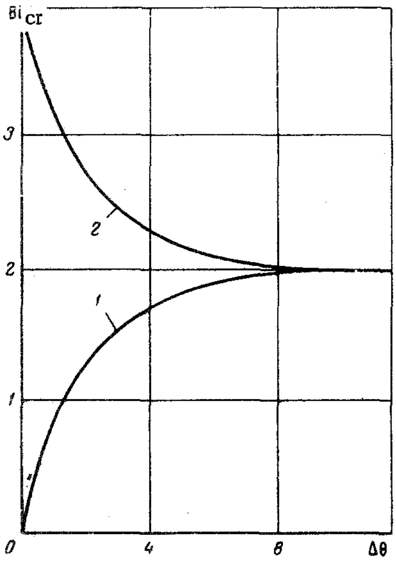

Fig. 1. Critical Biot number Bi_{cr} as a function of the temperature drop $\Delta\theta$: 1) exothermal reaction; 2) endothermal reaction.

For stipulated values of $\Delta\theta$ one determines the permissible value of the Biot number from the solution to the equation

$$c\,\mathrm{Bi}^2 - \frac{8\sigma\exp\left[-\frac{4\sigma}{\mathrm{Bi}(\sigma \pm 1)}\right]}{(\sigma \mp 1)^2} = 0, \quad (5)$$

where $c = |h|Ek\lambda_{eff}/R(T_a\alpha)^2$.

From this Bi one then determines the permissible tube radius

$$r_0 = \mathrm{Bi}\lambda_{eff}/\alpha.$$

It is to be noted that Eq. (5) has no solution when

$$\sigma c \geqslant 2\exp(-2) = 0{,}270671 \quad (6)$$

or

$$\mathrm{Bi}_{cr} < \frac{2\sigma}{\sigma \pm 1}. \quad (7)$$

In this case the stipulated temperature drop cannot be realized, regardless of the tube radius.

For real values of all parameters $\Delta\theta$ lies within 0-30. The calculations of Bi_{cr} as a function of $\Delta\theta$ are shown in Fig. 1.

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Characteristically, it is not possible to realize a zero temperature drop during an exothermal reaction. The maximum tube radius corresponds to small temperature drops during endothermal reactions, while $Bi_{cr} < 4$. When the temperature drop is large ($\Delta\theta > 10$), then $Bi_{cr} = 2$ for both exothermal and endothermal reactions.

NOTATION

T	is the local absolute temperature at a tube section;
T_c	is the temperature at the tube axis;
T_w	is the temperature at the tube wall;
T_a	is the ambient temperature;
$k = k_0 \exp(-E/RT_a)$	is the rate constant of the reaction;
k_0	is the cofactor of the exponential term in the rate constant;
h	is the thermal effect of the reaction;
R	is the universal gas constant;
E	is the activation energy of the reaction;
r	is the local radius;
r_0	is the inside radius of the tube;
$\rho = r/r_0$	is the dimensionless local radius;
λ_{eff}	is the effective radial thermal conductivity of the granular bed;
α	is the coefficient of heat transfer from bed to coolant;
$\theta = E(T-T_a)/RT_a^2$;	
$\delta = \lvert h \rvert E k r_0^2/\lambda_{eff} R T_a^2$	is the heat source (sink) parameter;
σ	are the roots of the characteristic equation (2);
$Bi = \alpha r_0/\lambda_{eff}$;	is the Biot number;
$\theta_c = E(T_c-T_a)/RT_a^2$;	
$\theta_w = E(T_w-T_a)/RT_a^2$.	

LITERATURE CITED

1. A. G. Gorelik, Inzh. Fiz. Zh., 21, No. 2, 265 (1971).